Fluctuational internal Josephson effect in topological insulator film

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Tunneling between opposite surfaces of topological insulator thin film populated by electrons and holes is considered. We predict considerable enhancement of tunneling conductivity by Cooper electron-hole pair fluctuations that are precursor of their Cooper pairing. Cooper pair fluctuations lead to critical behavior of tunneling conductivity in vicinity of critical temperature with critical index $\nu = 2$. If the pairing is suppressed by disorder the behavior of tunneling conductivity in vicinity of quantum phase transition is also critical with the index $\mu = 2$. The effect can be interpreted as fluctuational internal Josephson effect.

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I. INTRODUCTION

Cooper pairing of spatially separated electrons and holes was predicted in the system of semiconductor quantum wells more then thirty years ago[1]. Later it was observed in quantum Hall bilayer at total filling factor $\nu_{\mathrm{T}} = 1$ that can be presented as the system of spatially separated composite electrons and composite holes (see[2] and references therein). After graphene discovery Cooper pairing of Dirac electrons and holes in the structure of independently gated graphene layers has been proposed[3–5]. Recently possibility of Cooper pairing of Dirac electrons and holes was predicted[6,7] in thin film of topological insulator, new unique class of solids that has topologically protected Dirac surface states[8,9]. The electron-hole pairing in that system is the realization of topological superfluidity and hosts Majorana fermions on edges and vortices[7,10] that is the topic of extraordinary interest[11,12]. Also Cooper pairing can lead to a number of interesting physical effects including superfluidity[1], anomalous drag effect[13], nonlocal Andreev reflection[14] and prominent internal Josephson effect[15].

Internal Josephson effect manifests itself in colossal enhancement of tunneling conductivity at zero voltage bias and it has been observed in quantum Hall bilayer[16,17]. It is caused by coherence between electron and hole states which leads to a coherent tunnel current depending on the phase of electron-hole condensate. Microscopical and macroscopical description of internal Josephson effect was addressed in a number of interesting theoretical papers[18–22].

Cooper electron-hole pairs can appear above critical temperature as thermodynamic fluctuations. They lead to logarithmic divergence of a drag conductivity as function of a temperature[23,24] and a pseudogap formation in single-particle density of states of electrons and holes[25]. Manifestations of Cooper pair fluctuations in tunneling have not been consider previously. The pairing fluctuations can transfer coherent tunnel current and enhance tunneling conductivity above critical temperature. Since amplitude of pairing fluctuations increases in vicinity of critical temperature, as fluctuations of ordered state do for different phase transitions, one can expect the significant enhancement of tunneling conductivity. The described effect can be called fluctuational internal Josephson effect and in this work we develop its microscopic theory. This effect is considered in topological insulator thin film and its peculiarities in other realizations of electron-hole bilayer are discussed in Conclusions.

The rest of the paper is organized as follows. In Section 2 we briefly discuss the model and microscopical description of Cooper pair fluctuations. In Section 3 tunneling conductivity of topological insulator film is calculated and Section 4 is devoted to conclusions.

II. COOPER PAIR FLUCTUATIONS

We consider topological insulator thin film with independently gated surfaces occupied by excess electrons and holes. In the single-band approximation the Hamiltonian of the system is given by[6]

$$H_{\mathrm{eh}} = \sum_{\mathbf{p}} \xi_{\mathbf{p}} a^+_{\mathbf{p}} a_{\mathbf{p}} - \sum_{\mathbf{p}} \xi_{\mathbf{p}} b^+_{\mathbf{p}} b_{\mathbf{p}} + \\ + \sum_{\mathbf{p}\mathbf{p}'\mathbf{q}} U(\mathbf{q}) \Lambda^{\mathbf{p}+\mathbf{q},\mathbf{p}}_{\mathbf{p}'-\mathbf{q},\mathbf{p}'} a^+_{\mathbf{p}+\mathbf{q}} b^+_{\mathbf{p}'-\mathbf{q}} b_{\mathbf{p}'} a_{\mathbf{p}}. \tag{1}$$

Here $a_{\mathbf{p}}$ is annihilation operator for a electron on the surface with excess of electrons and $b_{\mathbf{p}}$ is annihilation operator for a electron on the surface with excess of holes[26]; $\xi_{\mathbf{p}} = v_{\mathrm{F}} p - E_{\mathrm{F}}$ is Dirac dispersion law in which v_{F} and E_{F} are velocity and Fermi energy of electrons and holes. We consider the balanced case since it is favorable for Cooper pairing and the pairing is sensitive to concentration mismatch of electrons and holes. $U(\mathbf{q})$ is screened Coulomb interaction between electrons and holes (see[6] for its explicit value) and $\Lambda^{\mathbf{p}+\mathbf{q},\mathbf{p}}_{\mathbf{p}'-\mathbf{q},\mathbf{p}'} = \cos(\phi_{\mathbf{p},\mathbf{p}+\mathbf{q}}/2)\cos(\phi_{\mathbf{p}',\mathbf{p}'+\mathbf{q}}/2)$ is angle factor that comes from the overlap of spinor wave functions of two-dimensional Dirac fermions.

For microscopical description of Cooper pair fluctuations we introduce Cooper propagator. It corresponds to the two-particle vertex function in the Cooper channel[27]

and satisfies the Bethe-Salpeter equation depicted on Fig. 1 (a). In Bardeen-Cooper-Schrieffer (BCS) approximation its solution can be presented in the form

$$\Gamma_{\mathrm{c}}^{\mathrm{R}}(\omega)=\frac{U'}{1-U'\Pi_{\mathrm{c}}^{\mathrm{R}}(\omega)}, \tag{2}$$

where U' is the Coulomb coupling constant[28] and $\Pi_{\mathrm{c}}^{\mathrm{R}}(\omega)$ corresponds to electron-hole bubble diagram that can be interpreted as Cooper susceptibility of the system. Direct calculation of this diagram leads to

$$\Gamma_{\mathrm{c}}^{\mathrm{R}}(\omega)=\frac{1}{\nu_{\mathrm{F}}}\frac{1}{\ln\frac{T}{T_0}+\Psi(\frac{1}{2}-i\frac{\omega}{4\pi T}+\frac{\gamma}{2\pi T})-\Psi(\frac{1}{2})}. \tag{3}$$

Here $\gamma=(\gamma_a+\gamma_b)/2$ is disorder caused Copper pair decay rate equal to half-sum of phenomenological introduced decay rates of electrons and holes; T_0 is the critical temperature of electron-hole pairing without disorder; ν_{F} is the density of states of electrons and holes on the Fermi level; $\Psi(x)$ is the digamma function. In the absence of disorder $\Gamma_{\mathrm{c}}(\omega)=0$ at the critical temperature T_0 indicating Cooper instability of the system against Cooper pairing. Critical temperature for disordered system T_{d} satisfies the following equation

$$\ln\frac{T_{\mathrm{d}}}{T_0}+\Psi\left(\frac{1}{2}+\frac{\gamma}{2\pi T}\right)-\Psi\left(\frac{1}{2}\right)=0. \tag{4}$$

This equation has nontrivial solution if $\gamma<\gamma_{\mathrm{c}}$, where $\gamma_c=0.89T_0$ is the critical Cooper pair decay value. In opposite case the pairing is suppressed by disorder. The value γ_{c} corresponds to quantum critical point at zero temperature.

In a vicinity of the critical temperature T_{d} the expression for Cooper pair propagator (3) can be approximated in following way

$$\Gamma_{\mathrm{c}}^{\mathrm{R}}(\omega)=\frac{1}{\nu_{\mathrm{F}}}\frac{4\pi T_{\mathrm{d}}}{4\pi(T-T_{\mathrm{d}})-i\omega\Psi'(\frac{1}{2}+\frac{\gamma}{2\pi T_{\mathrm{d}}})}. \tag{5}$$

In a vicinity of quantum phase transition at zero temperature Cooper pair propagator is given by

$$\Gamma_{\mathrm{c}}^{\mathrm{R}}(\omega)=\frac{1}{\nu_{\mathrm{F}}}\frac{2\gamma_{\mathrm{c}}}{2(\gamma-\gamma_{\mathrm{c}})-i\omega}. \tag{6}$$

III. TUNNELING CONDUCTIVITY

The tunneling of electrons between the opposite surfaces of topological insulator thin film with conserving momentum can be described by the following Hamiltonian

$$H_{\mathrm{T}}=T+T^{+}=\sum_{\mathbf{p}}\left(tb_{\mathbf{p}}^{+}a_{\mathbf{p}}+t^{*}a_{\mathbf{p}}^{+}b_{\mathbf{p}}\right), \tag{7}$$

where t is the tunneling amplitude. For a calculation of the tunneling conductivity we use linear response theory in which the tunneling conductivity $\sigma_{\mathrm{T}}(V)$ at finite voltage bias V can be presented in the form of Kubo formula[29]

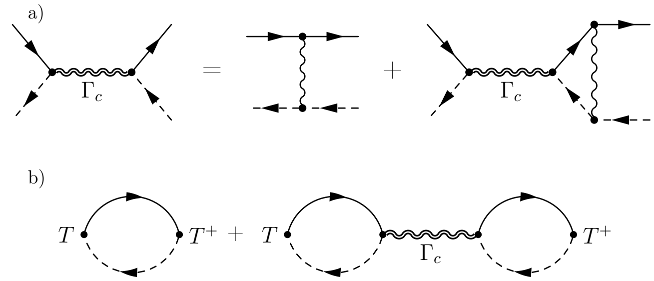

FIG. 1. a) Diagrammatic representation of the Bethe-Salpeter equation for the Cooper propogator Γ_{c}; b) Feynman diagrams for a tunnel conductivity. Solid (dashed) line corresponds to electrons on the surface of TI film with excess of electrons (holes).

$$\sigma_{\mathrm{T}}(V)=\frac{e^2}{h}\frac{4\pi}{eV}\,\mathrm{Im}[\chi_{\mathrm{T}}^{\mathrm{R}}(eV)], \tag{8}$$

where the retarded response function $\chi_{\mathrm{T}}^{\mathrm{R}}(\omega)$ can be obtained by analytical continuation $i\Omega_n\to\omega+i\delta$ of $\chi_{\mathrm{T}}^{\mathrm{M}}(i\Omega_n)$ that is given by

$$\chi_{\mathrm{T}}^{\mathrm{M}}(i\Omega_n)=-\frac{1}{2\beta}\int_{-\beta}^{\beta}d\tau\; e^{i\Omega_n\tau}\langle T_{\mathrm{M}}T(\tau)T^{+}(0)\rangle. \tag{9}$$

Here T_{M} is time-ordering symbol for imaginary time τ and $\Omega_n=2\pi nT$ is bosonic Matsubara frequency. In the system of noninteracting electrons and holes the $\chi_{\mathrm{T}}^{\mathrm{M}}(i\Omega_n)$ corresponds to the first diagram on the Fig 1 (b) leading to $\chi_{\mathrm{T}}^{\mathrm{R}}(\omega)=|t|^2\Pi_{\mathrm{c}}^{\mathrm{R}}(\omega)$. Hence the tunneling conductivity for noninteracting electrons and holes σ_{T0} is given by

$$\sigma_{\mathrm{T0}}(V)=\frac{e^2}{h}\frac{4\pi|t|^2}{eV}\,\mathrm{Im}[\Pi_{\mathrm{c}}^{R}(eV)]. \tag{10}$$

Its value $\sigma_{\mathrm{T0}}^{\mathrm{m}}(\gamma,T)$ at zero bias is given by

$$\sigma_{\mathrm{T0}}^{\mathrm{m}}(\gamma,T)=\frac{2\pi e^2}{h}\frac{\nu_{\mathrm{F}}|t|^2}{2\pi T}\Psi'\left(\frac{1}{2}+\frac{\gamma}{2\pi T}\right), \tag{11}$$

and at low temperatures $T\ll\gamma$ it transforms to

$$\sigma_{\mathrm{T0}}^{\mathrm{m}}(\gamma,0)=\frac{2\pi e^2}{h}\frac{\nu_{\mathrm{F}}|t|^2}{\gamma}. \tag{12}$$

Introduction of the electron-hole Coulomb interaction in the ladder approximation leads to the additional term for tunneling conductivity that corresponds to the second diagram on the Fig 2 (b). Infinite series of electron-hole scattering diagrams in Cooper channel correspond to Cooper propagator hence the second term can be attributed to the pairing fluctuations. The tunneling conductivity for interacting electrons and holes σ_{Tc} is given by

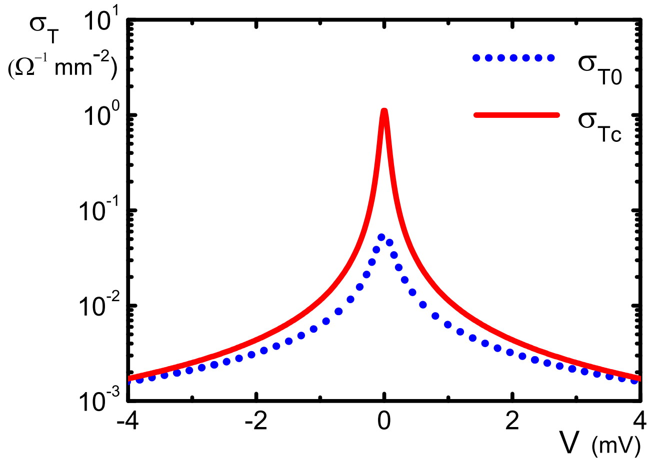

FIG. 2. (Color online) Tunneling conductivity $\sigma_{\rm T}$ as a function of bias voltage V for noninteracting (dashed line) and interacting (solid line) electrons and holes for $\gamma = 0.2$ K and $T = 0.2$ K.

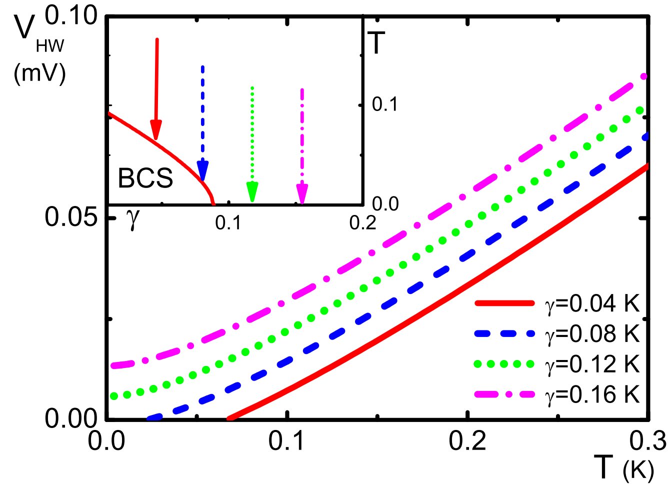

FIG. 3. (Color online) Half-width of the tunnel conductivity peak $V_{\rm HW}$ for interacting electrons and holes as function of temperature T for different values of Cooper pair decay rate γ. Inset: The phase diagram of the system in which BCS denotes the pairing state. The arrows on the phase diagram correspond to the dependencies on Fig. 3 and Fig. 4.

$$\sigma_{\rm Tc}(V) = \frac{e^2}{h}\frac{4\pi|t|^2}{eV}\,{\rm Im}\left[\frac{\Pi^{\rm R}_{\rm c}(eV)}{1-U'\Pi^{R}_{\rm c}(eV)}\right]. \qquad (13)$$

The denominator of (13) coincides with that in the Cooper propagator (2) and tends to zero in a vicinity of critical temperature $T_{\rm d}$. Hence the Cooper pair fluctuations lead to critical behavior of tunneling conductivity in vicinity of critical temperature and quantum critical point. In vicinity of critical temperature $T_{\rm d}$ tunneling conductivity at zero bias is given by

$$\sigma^{\rm m}_{\rm Tc} = \sigma^{\rm m}_{\rm T0}(\gamma, T_{\rm d})\frac{1}{(\nu_{\rm F}U')^2}\frac{T_{\rm d}^2}{(T-T_{\rm d})^2}. \qquad (14)$$

In vicinity of critical temperature it diverges as $\sigma_{\rm T} \sim (T-T_{\rm d})^{-\nu}$ with the critical index $\nu = 2$. At zero temperature in the vicinity of the quantum phase transition at $\gamma = \gamma_{\rm c}$ the tunneling conductivity is given by

$$\sigma^{\rm m}_{\rm Tc} = \sigma^{\rm m}_{\rm T0}(\gamma, 0)\frac{1}{(\nu_{\rm F}U')^2}\frac{\gamma_{\rm c}^2}{(\gamma-\gamma_{\rm c})^2}. \qquad (15)$$

It diverges as $\sigma_{\rm T} \sim (\gamma-\gamma_{\rm c})^{-\mu}$ with critical index $\mu = 2$. The formulas (14) and (15) are the main result of the article

For numerical calculation of the tunneling conductivity at finite voltage bias we use formulas (3),(13) and the following set of the parameters[6] $T_0 = 0.1$ K, $\mu = 5$ meV, $t = 2$ μeV. The phase diagram of the system is presented on the inset of Fig.3. If Cooper pair decay rate exceeds value $\gamma_{\rm c} = 0.09$ K then the electron-hole pairing is suppressed by disorder.

Calculated tunneling conductivity for $\gamma = 0.2$ K and $T = 0.2$ K is presented on Fig. 2. Due to restrictions connected with energy and momentum conservation the dependence has prominent peak. Such peak was predicted and observed also in electron-electron bilayer[30,31] and it is peculiarity of tunneling between two two-dimensional systems. Coulomb interaction between electrons and holes considerably enhances tunneling conductivity but does not change qualitatively its dependence on external bias. Half-width and height of the peak for interacting electrons and holes are presented on Fig. 3 and Fig. 4, respectively. For $\gamma < \gamma_{\rm c}$ Coulomb interaction leads to the critical behavior of the tunneling conductivity that we interpret as manifestations of Cooper pair fluctuations. In vicinity of the critical temperature height of the peak diverges with the critical index $\nu = 2$ that agrees with the analytic results and width of the peak linearly tends to zero. If Cooper pair decay exceeds critical value $\gamma > \gamma_{\rm c}$ Coulomb interaction considerably enhances the height and leads to reduction of the width but does not lead to any singularities. The peak becomes more prominent with decreasing of decay and temperature as it does in the model of noninteracting electrons and holes.

IV. CONCLUSIONS

We have shown that electron-hole Coulomb interaction considerably enhances tunneling conductivity in electron-hole bilayer even when the Cooper pairing is suppressed by disorder. The opposite situation takes place in electron-electron bilayer that also can be realized in semiconductor quantum well structure, in graphene double layer system and in a film of topological insulator. Coulomb interaction gives contribution to decay of electrons that was analyzed in[32] and to additional series of diagrams for the tunneling conductivity. We treated the additional diagrams in ladder approximation (See Fig.2-

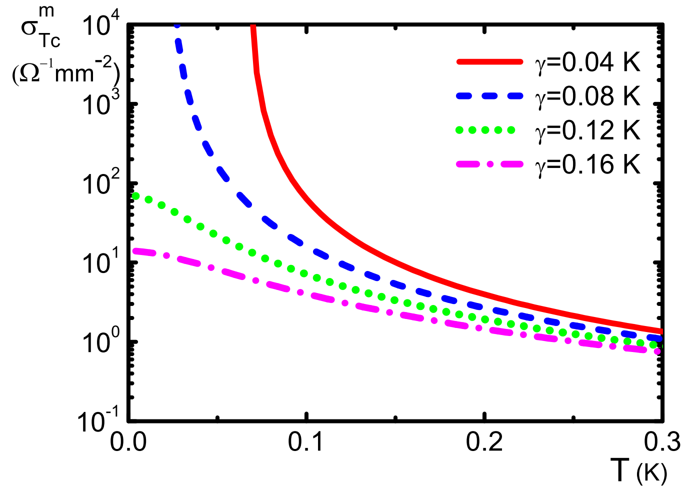

FIG. 4. (Color online) The height of tunnel conductivity peak $\sigma^{\rm m}_{\rm Tc}$ for interacting electrons and holes on temperature T for different values of Cooper pair decay γ.

b). If they are omitted the tunneling conductivity at zero temperature and at finite bias V is given by[31,32]

$$\sigma^{\rm ee}_{\rm T0} = g t_{\rm A}\frac{2\pi e^2}{h}\frac{\nu_{\rm F}|t|^2}{\gamma}\frac{4\gamma^2}{4\gamma^2+(eV)^2}, \tag{16}$$

where g is degeneracy factor of electrons and $t_{\rm A}$ is additional factor that depends on internal nature of electrons[33]. The dependence of tunnel conductivity on external bias contains prominent peak which becomes more prominent with decreasing of decay rate γ. If the Coulomb interaction is treated in ladder approximation the tunneling conductivity is given by

$$\sigma^{\rm ee}_{\rm Tc} = g t_{\rm A}\frac{2\pi e^2}{h}\frac{\nu_{\rm F}|t|^2}{\gamma}\frac{4\gamma^2}{4\gamma^2+(1+(\nu_{\rm F}U')^2)(eV)^2}. \tag{17}$$

For electron-electron bilayer Coulomb interaction does not influences the height of the peak and leads to decreasing of peaks width which is insignificant even in case of strong interaction $\nu_{\rm F}U' \sim 1$. The roles of the interlayer Coulomb interaction in electron-electron bilayer and electron-hole bilayer are drastically different because the correction to the tunneling conductivity of the primer is caused by the scattering diagrams in particle-antiparticle channel and the correction to the one of the latter is caused by the diagrams in particle-particle Cooper channel containing instability.

The mean field theory we use here for description of fluctuational internal Josephson effect does not account large scale fluctuation of phase of Cooper pair condensate. In two-dimensional superfluids phase fluctuations destroy long-range coherence and the transition to paired state at critical temperature $T_{\rm d}$ calculated within mean field theory is smoothed. Moreover the transition to superfluid state is Berezinskii-Kosterlitz-Thouless transition[34,35] that corresponds to dissociation of vortex-antivortex pairs and which temperature is below $T_{\rm d}$. Hence large scale phase fluctuations of Cooper pair condensate can smooth critical behavior of tunnel conductivity we predict here. But if the size of the system L is comparable with coherence length of Cooper pair fluctuations $l_{\rm c} \approx \hbar v_{\rm F}/T_0$ the phase fluctuations are unimportant and mean field theory is well applicable. For $T_0 = 0.1$ K the coherence length of Cooper pair fluctuations is of order $l_{\rm c} \sim 10$ mkm and we conclude that the developed microscopical theory is applicable for samples of the corresponding size.

The model we use here implies conservation of the momentum of tunneling electron. If the momentum is not conserved the tunneling process creates electron-hole pair with nonzero total momentum of order $l_{\rm T}^{-1}$. Here $l_{\rm T}$ is character length at which tunneling matrix matrix element t can be considered as constant. Cooper pair is formed by electron and hole with opposite momenta and the Cooper instability is smoothed if $l_{\rm c} \gg l_{\rm T}$. For tunneling between opposite surfaces of topological insulator thin film of high crystalline quality momentum conservation can be achieved to remarkable degree.

We have investigated the manifestations of Cooper electron-hole pairing fluctuations in thin film of topological insulator on tunneling between its opposite surfaces. The effect takes place in other realizations of electron-hole bilayer but each system has its own peculiarities.

Dirac points in graphene are situated in corners of first Brillouin zone. Electron-hole pairing was predicted[3–5] in the system of two independently gated graphene layers separated by dielectric film. In that case orientations of the graphene lattices are uncorrelated. The distance between Dirac points of different layers in momentum space is of order a_0^{-1}, where a_0 is lattice constant of graphene. The tunneling of electrons between Dirac points is possible if $l_{\rm T} \sim a_0$ that corresponds to tunneling through impurity states or other defects. The condition $l_{\rm c} \gg l_{\rm T}$ is well satisfied and the critical behavior of tunneling conductivity in double layer graphene system is considerably smoothed.

Recently anomalies in drag effect in semiconductor double well structure that contains spatially separated electrons and holes were observed[36,37]. The analysis of results shows that the observed anomalies can be caused by electron-hole pairing not in BCS regime but rather in regime of BCS-BEC crossover[38,39]. In that regime electron-hole pairing fluctuations also should increase tunneling conductivity of the system but quantitative theory of the effect is interesting and challenging problem. The developed here microscopical theory of fluctuation internal Josephson effect is applicable for that system if the pairing is realized in BCS regime.

Internal Josephson effect has been observed in quantum Hall bilayer at total occupation factor $\nu_{\rm T} = 1$. But above critical temperature the dependence of tunneling conductivity on external bias does not contain any peak due to non Fermi-liquid behavior of composite electrons and holes[40–43]. Thus fluctuational effects in the system

are more complicated ones and need separate investigation. It should be noted that critical behavior of tunneling conductivity has not been observed yet in experiments in that system.

We have considered influence of electron-hole Cooper pair fluctuations that are precursor of Cooper pairing in topological insulator film on tunneling between its opposite surface. Cooper pair fluctuations lead to critical behavior of tunneling conductivity in vicinity of critical temperature with critical index $\nu = 2$. If pairing is suppressed by disorder the behavior of tunneling conductivity in vicinity of quantum critical point at zero temperature is also critical with critical index $\mu = 2$. The effect can be interpreted as fluctuational Josephson effect.

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